especial interest to readers of the MONTHLY because three of them are presidential addresses to the "Mathematical Association." The title of the book is "The organization of thought." It is published by Williams and Norgate, London.

The D. Van Nostrand Company has recently published a book on "Recreations in mathematics," by H. E. Licks. It has chapters on arithmetic, algebra, geometry, trigonometry, analytic geometry, calculus, astronomy and the calendar, mechanics and physics, and an appendix. It is not so extensive as the well-known "Mathematical recreations and essays" by W. W. R. Ball, but it contains considerable material not to be found in that standard work. The eight pages devoted to the cell of the honey bee will be new to many readers.

PROBLEMS FOR SOLUTION.

SEND ALL COMMUNICATIONS ABOUT PROBLEMS TO B. F. FINKEL, Springfield, Mo.

2689. Proposed by E. V. HUNTINGTON, Cambridge, Mass.

Show that the maximum value of

$$y = \frac{\sin x \cos (x + \varphi)}{\cos (x + \theta) \sin (x + \varphi + \theta)}$$

is $y_1 = (\cos \theta - \rho)/(\cos \theta + \rho)$, where $\rho = \sqrt{\sin^2 \varphi - \sin^2 \theta}$.

This problem was suggested to the proposer by a professor of civil engineering, and has important applications in the theory of conjugate stresses.

Note.—It may facilitate the work to let $\xi = 2x + \varphi + \theta$.

2690. Proposed by E. V. HUNTINGTON, Cambridge, Mass.

Find the maximum value of

$$y = \frac{\sin x \cos (x + \varphi)}{\cos (x + \theta) \sin (x + \beta + \theta)}.$$

2691. Proposed by ROGER A. JOHNSON, Hamline University.

Show by purely geometric methods, without the use of the calculus, that the envelope of all circles whose centers are on a fixed circle and which touch a fixed diameter of that circle is a two-arched epicycloid. (Cf. Calculus problem, 423.)

2692. Proposed by J. L. RILEY, Stephenville, Texas.

A cube is cut at random by a plane, what is the chance that the section is a hexagon?

2693. Proposed by W. F. HARLOW, Portland, Oregon.

A cow is tethered with a rope, length l, to a peg on the opposite side of a wall, height h, the peg being at a distance a from the wall. Find the area over which the cow can graze.

2694. Proposed by N. P. PANDYA, Sojitra, India.

Find the locus of the centroid of a triangle, whose vertex lies on a given parabola, whose base of given length is a segment of a given straight line of unlimited length, and one of whose base angles is known.

2695. Proposed by FRANK IRWIN, University of California.

A positive number, which for convenience we will write as a fraction, a/b, is developed into a continued fraction,

$$a_1 \frac{1}{-a_2} \frac{1}{-a_3} - \cdots,$$

by the following process:

$$a = a_1b - r_1, \qquad b = a_2r_1 - r_2, \qquad r_1 = a_3r_2 - r_3, \text{ etc.}$$

Here $a_1, a_2, \cdots$ are positive integers, and, if we suppose a, b to have been taken positive, $b > r_1 > r_2 > \cdots > 0$. Show that if a/b be but slightly larger than a positive integer or zero, the numerators (as likewise the denominators) of the successive convergents will for a time be in arithmetical progression, and determine how long this phenomenon will continue.

2696. Proposed by L. E. LUNN, Heron Lake, Minnesota.

An air pipe 18 inches in diameter passes diagonally through a room from one lower corner to the opposite upper corner leaving through elliptical openings in the floor and ceiling, so that the ellipses are tangent to two boundaries of the floor and to the two opposite boundaries of the ceiling. If the room is 10 x 12 x 8, find the remaining cubic capacity of the room.

2697. Proposed by H. S. UHLER, Yale University.

Show how to reduce the left-hand members of the following identities to their respective right members:

$$\sin^2 (x + \tfrac{1}{2}y) - \sin (x + \tfrac{3}{2}y) \sin (x - \tfrac{1}{2}y) = \sin^2 y,$$

$$\sin (x + y) \sin (x + \tfrac{1}{2}y) - \sin x \sin (x + \tfrac{3}{2}y) = \sin \tfrac{1}{2}y \sin y,$$

$$\sin x \sin (x + \tfrac{1}{2}y) - \sin (x - \tfrac{1}{2}y) \sin (x + y) = \sin \tfrac{1}{2}y \sin y.$$

2698. Proposed by WARREN WEAVER, Throop College of Technology, Pasadena, California.

An urn contains N balls numbered from 1 to N. Of these n are drawn out and are arranged linearly according to the numbers on each. A certain ball is observed to be the kth in this line. What is the most probable number written on this ball?

SOLUTIONS OF PROBLEMS.

490 (Algebra). Proposed by HENRY HEATON, Atlantic, Iowa.

Show that $\sin 3° = \frac{1}{16}(\sqrt{30} + \sqrt{10} - \sqrt{6} - \sqrt{2}) + \frac{1}{8}(\sqrt{5 + \sqrt{5}} - \sqrt{15 + 3\sqrt{5}})$.

SOLUTION BY S. E. RASOR, The Ohio State University.

Since $3° = 12° - 9°$, $12° = 30° - 18°$, and, for $\theta = 18°$, $2\theta = 90° - 3\theta$, the sine and the cosine of 9°, 12°, 18°, and thus sin 3° may be found as follows:

We have

$$\sin 2\theta = 2 \sin \theta \cos \theta = \sin (90° - 3\theta) = \cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta,$$

$$4 \sin^2 \theta + 2 \sin \theta = 1,$$

and

$$\sin 18° = \tfrac{1}{4}(\sqrt{5} - 1), \quad \cos 18° = \tfrac{1}{4}\sqrt{10 + 2\sqrt{5}}.$$

Also from the identities, $\sin \frac{1}{2}A \pm \cos \frac{1}{2}A = \pm \sqrt{1 \pm \sin A}$ for $A = 18°$, we have

$$\sin 9° = \tfrac{1}{4}(\sqrt{3 + \sqrt{5}} - \sqrt{5 - \sqrt{5}}), \qquad \cos 9° = \tfrac{1}{4}(\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}).$$

Also,

$$\sin 12° = \sin (30° - 18°) = \sin 30° \cos 18° - \cos 30° \sin 18° = \tfrac{1}{8}(\sqrt{10 + 2\sqrt{5}} - \sqrt{15} + \sqrt{3}),$$

$$\cos 12° = \tfrac{1}{8}(\sqrt{30 + 6\sqrt{5}} + \sqrt{5} - 1).$$

Therefore,

$$\sin 3° = \sin (12° - 9°) = \sin 12° \cos 9° - \cos 12° \sin 9°$$

$$= \left(\frac{\sqrt{10 + 2\sqrt{5}} - \sqrt{15} + \sqrt{3}}{8}\right)\left(\frac{\sqrt{3 + \sqrt{5}} + \sqrt{5 - \sqrt{5}}}{4}\right) - \left(\frac{\sqrt{30 + 6\sqrt{5}} + \sqrt{5} - 1}{8}\right)\left(\frac{\sqrt{3 + \sqrt{5}} - \sqrt{5 - \sqrt{5}}}{4}\right)$$